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by

Elmor L. Peterson

(October 1, 1976 through March 31, 1979)

GENERALIZED GEOMETRIC PROGRAMMING
WITH APPLICATIONS

During the grant period my research efforts have been concentrated

in five major directions: (1) the effective formulation and study of two

important optimization problems and two important equilibrium problems

(as generalized geometric programming problems), (2) an investigation of

the relations between suboptimization and parameter deletion, including the

relations between ordinary duality, geometric duality, and Rockafellar

duality, (3) an investigation of the relations between the fixed point

problem and the geometric complementarity problem (a generalization of

the ordinary complementarity problem), (4) an extension of the classical

existence theorems for both the fixed point problem and the variational

inequality problem, to deal with the geometric complementarity problem,

(5) the preparation of a book that unifies and contrasts ordinary program-

ming theory, geometric programming theory, parametric programming theory,

ordinary complementarity theory, geometric complementarity theory, fixed

point theory, and variational inequality theory; while showing how each

of these theories supplies different insights into various important

optimization problems and equilibrium problems.

The two important optimization problems and the two important equili-

brium problems alluded to in (1) are: (i) generalized Fermat problems that

arise in the optimal location of facilities, (ii) discrete optimal control problems that have linear dynamics and arise in many different contexts, (iii) economic equilibrium problems in which spatial dependencies are accounted for, (iv) traffic equilibrium problems in which multimodal phenomena and intersection phenomena are accounted for. The first draft of a comprehensive treatment [11] of problem (i) is nearing completion; and although a treatment of problem (ii) has been accepted for publication as [2], it is still undergoing extensive revisions prior to its appearance. The treatment of problem (iii) originally described in [8] has been expanded from a "partial equilibrium model" to a "complete equilibrium model" and will be published as [9]. Moreover, an algorithmic treatment of the special affine case has been accepted for publication as [1]. Problem (iv) has recently been studied via the methods alluded to in (4), but a publication describing the results has not yet been drafted.

The relations between suboptimization and parameter deletion mentioned in (2) have been published as [5]. Moreover, the relations between ordinary duality and geometric duality have been accepted for publication as [6], while the relations between geometric duality and Rockafellar duality have been accepted for publication as [7].

The relations between the fixed point problem and the geometric complementarity problem refered to in (3) have been submitted for publication as [4]. In addition to their significance for geometric programming, they unify the Eaves and Kojima fixed point representations of the ordinary complementarity problem.

The extension of the classical existence theorems for both the fixed point problem and the variational inequality problem mentioned in (4) was

n... ...sary because the existence theorems of Brouwer-Kakutani-Eilenberg-
M... ...omery, Leray-Schauder, and Hartman-Stampacchi-Saigal require boundedness
of ... ...ch the mapping's domain and its individual image sets--hypotheses
th... ...re too strong to deal effectively with the geometric complementarity
pr... ...em because of its generally conical-like domain and the possible
un... ...dedness of its image sets at certain domain boundary points. The
re... ...s obtained so far extend such existence theorems by the introduction
of ... ...theses that essentially compactify the unbounded domain and eliminate
so... ...f its troublesome boundary points via coercivity conditions tailored
to ... ... specific nature of the geometric complementarity problem. Even in
th... ...cial case of the ordinary complementarity problem (whose domain is
th... ...-negative orthant), the results are stronger than the corresponding
re... ... previously obtained by More', Saigal and others. In the more
ge... . case, the results provide new existence theorems for the equilibrium
pr... ...s (iii) and (iv) mentioned as part of (1). They also provide new
ex... ...e theorems for certain non-monotone electrical network problems
th... ...ght conceivably arise with the use of some non-monotone electronic
de... Although all of these results will soon appear in the form of a
Ph... ...esis [3] by Shu-Cherng Fang (a student who has received some
fi... ...l support from the grant during the past year), they will eventually
be ... ...shed as several different papers by Mr. Fang and the author.
... book [10] refered to in (5) will probably require at least two
mo... ...s for completion. It will present a very comprehensive treatment
of ... ...sted topics; and close to one half of it now exists in the form of
ty... ...ture notes.
... ...or the completed papers in the bibliography have been issued and

circulated in the form of AFOSR scientific reports or journal reprints.

The uncompleted papers will eventually be handled in a similar fashion.

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